Topology

Semestral Examination

26th November 2012

Instructions: All questions carry equal marks.

1. Is the product space $[0,1] \times [0,1)$ homeomorphic to the product $[0,1) \times [0,1)$? Justify your answer.

2. Let X denote the set of *diadic rationals*, i.e., the rational numbers of the type $\frac{m}{2^n}$ for integers m and n. Let Y denote the remaining rational numbers and Z denote the set of irrational numbers. Let τ denote the topology on the set of real numbers $\mathbb{R}$ generated by the following sets:

- **(i):** every open U in the standard topology
- **(ii):** the sets X and Y.
- **(iii):** sets of the type $\{z\} \cup \{w \in X \cup Y| \quad |w - z| < \delta\}$ for $\delta > 0$ and $z \in Z$.

Prove that $\mathbb{R}$ is connected in this topology.

3. Define a topology τ on $[-1,1]$ as follows. A subset U of $[-1,1]$ is open if and only if either U contains $(-1,1)$ or U does not contain 0. Let X denote the corresponding topological space. Answer following questions with justifications.

- **(a):** Is X compact?
- **(b):** Is X connected?
- **(c):** Is X first countable?
- **(d):** Is X second countable?
- **(e):** Is X metrizable?

4. If $p_1 : X \to Y$ and $p_2 : X_1 \to Y_1$ are covering maps, then prove that $p_1 \times p_2$ is also a covering map.

5. What is the fundamental group of $S^1 \times \mathbb{R}$? Justify your answer.